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Attitude Control of a Quad-rotor System Using an Acceleration-based Disturbance Observer: An Empirical Approach*

Seung H. Jeong, Seul Jung, and M. Tomizuka

Abstract— **This paper presents the disturbance observer design for attitude control of a quad-rotor system. The quad-rotor system is approximated by an inertial system of which an acceleration term is dominant in dynamic behaviors. Acceleration-based disturbance observer (AbDOB) is designed to reject any disturbances for robust attitude control of the system. Acceleration is measured by a gyro sensor and used to estimate control input to estimate the disturbance. Then disturbance is cancelled with the estimated disturbance. Experimental studies are demonstrated to confirm the proposed control scheme.**

I. Introduction

Research on autonomous unmanned aerial vehicles (AUAV) has been quite active in military service as well as public applications. They can navigate autonomously to the designated areas and conduct tasks in the target point. AUAV is aimed to perform missions such as surveillance and monitoring tasks to explore dangerous or unknown areas that human beings cannot easily access.

A majority of AUAVs to perform missions in long range has the conventional take-off and landing (CTOL) structure such that long pavement is required to take off and land. AUAV with CTOL structure is suitable for long distance missions.

Meanwhile, there is a need for a short range mission, especially in the limited urban area where quad-rotors can take-off and land easy. The vertical take-off and landing structure (VTOL) allows AUAV to perform missions in the complicated environment surrounded by buildings or packed by cars in the highways.

Quad-rotor systems are gaining high attraction from researchers in the field of control and robotics communities since they are suitable for undertaking missions in urban areas. Different from single-rotor systems, quad-rotor systems have stable hovering posture and more power to carry objects since they have four rotors.

In addition, quad-rotor systems can have better maneuverability in omni-direction which is a superior advantage of taking missions in complicated areas.

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Seung H. Jeong and Seul Jung are with the Department of Mechatronics Engineering, Chungnam National University, Daejeon, 305-764 KOREA (phone: +82-42-821-6876; fax: +82-42-823-4919; e-mail: jungs@ cnu.ac.kr).

M. Tomizuka is with the Department of Mechanical Engineering, University of California, Berkeley, CA 94720-1740 USA, (e-mail: tomizuka@me.berkeley.edu).

Therefore, quad-rotor systems are used as a utility vehicle for public usages such as monitoring traffic conditions in the highway or congestive roads and investigating fire in the building.

As demand on quad-rotor systems increases, research on quad-rotor systems becomes active. Since the quad-rotor system is an under-actuated system that four rotors control six variables such as translation of x, y, z and rotation of ϕ, θ, ψ . Control of four variables of the rotation, ϕ, θ, ψ and the altitude z turns out to be the attitude control which is the most important characteristic of quad-rotor systems [1, 2].

In the literature, various control algorithms of quad-rotor system are presented. A nonlinear sliding mode control approach [3] and a simple linear PD control approach [4] are presented. Both nonlinear and linear control methods are compared [5]. An inverse dynamics control method is applied to improve control performances [6]. Neural network control approach has been presented [7, 8] along with other control approach [9-11].

Real-time controls of physical quad-rotor system are presented [12-15]. Applying quaternion-based feedback control scheme to quad-rotor system is presented [12]. Aggressive maneuvering control has been demonstrated [14]. Quad-rotor systems are designed differently to have both flying and driving capabilities [16, 17]. Autonomous navigation based on localization with the help of sensor fusion [18] and vision information [19-21]. Landing control using optical flow is presented [22].

Quad-rotor systems suffer from inherent and external disturbance. Such disturbances have significant effects on the stability of the quad-rotor system. Therefore, in this paper, an acceleration-based disturbance observer (AbDOB) is presented as a robust control algorithm. Acceleration signals are measured by sensors and used to estimate the control input to the system. Then the disturbance is estimated by the difference between the nominal control input and the estimated control input. That difference cancels out the disturbance. Experimental studies are conducted to demonstrate the performance by the AbDOB.

II. Quad-rotor system

A. Quad-rotor Dynamics Equations

The quad-rotor system is defined in the inertial coordinate system as shown in Fig. 1. Two rotors form a pair and each pair rotates in the opposite direction. Each rotor generates a thrust force f_i . Forces f_1 and f_3 form a pair and forces f_2 and f_4 form the other pair.

The generalized variable of the quad-rotor is

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$$q = (x, y, z, \phi, \theta, \psi)^T \quad (1)$$

where $q_1 = (x, y, z)^T$ is the position vector from the inertial frame to the body frame at the center of gravity and $q_2 = (\phi, \theta, \psi)^T$ is the rotational angles of roll, pitch, and yaw.

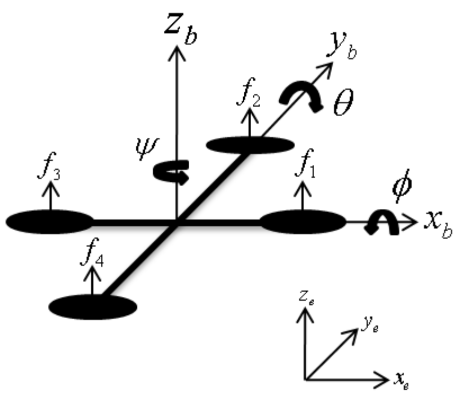

Fig. 1 Coordinate of Quad-rotor system

The quad-rotor dynamics is obtained from Euler-Lagrange equations with external forces.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad (2)$$

where L is the Lagrangian equation, τ_i is the ith torque such that $\tau = [F^T, T_{ROT}{}^T]^T$ where $F \in R^{3\times1}$ is the translational force vector and $T_{ROT} \in R^{3\times1}$ is the rotational torque vector, respectively.

The translational force can be represented as $F = R_T \hat{F}$ with a transformation matrix R and total thrust control input f_{th} as shown below.

$$\hat{F} = \begin{pmatrix} 0 \\ 0 \\ f_{th} \end{pmatrix} \quad (3)$$

$$R_T = \begin{pmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & s\theta s\psi c\phi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix} \quad (4)$$

where $ci = \cos(i), si = \sin(i)$.

The rotational torque vector T contains rotational torque elements. Each element is defined with forces of each rotor and the length between rotor and center of gravity.

$$T_{ROT} = \begin{bmatrix} T_\phi \\ T_\theta \\ T_\psi \end{bmatrix} \cong \begin{bmatrix} C_t(f_1 - f_2 + f_3 - f_4) \\ (f_3 - f_1)l \\ (f_2 - f_4)l \end{bmatrix} \quad (5)$$

where C_t is conversion factor from force to reaction torque and l is the length between the center of gravity and the rotor.

A simplified quad-rotor motion equation can be obtained under the assumption that gyroscopic and centrifugal terms are neglected.

$$m\ddot{q}_1 - mg\bar{z} = R\hat{F} \quad (6)$$

$$\bar{I}\ddot{q}_2 = T_{ROT} \quad (7)$$

where $\bar{I}$ is the matrix of the moment of inertia, $\bar{I} = diag\{I_{xx}, I_{yy}, I_{zz}\}$ where I_{xx}, I_{yy}, I_{zz} are the moment of inertia of each axis and $\bar{z}$ is the gravitational vector. The detailed dynamic equations are described as

$$\begin{aligned} m\ddot{x} &= u_{th}(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) \\ m\ddot{y} &= u_{th}(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi) \\ m\ddot{z} &= u_{th}\cos\theta\cos\phi - mg \\ I_{xx}\ddot{\psi} &= u_\phi \\ I_{yy}\ddot{\theta} &= u_\theta \\ I_{zz}\ddot{\phi} &= u_\psi \end{aligned} \quad (8)$$

where m is the mass of the system, g is the gravitational acceleration. For attitude control, a reduced dynamic equation is used such as

$$\begin{aligned} m\ddot{z} &= u_{th}\cos\theta\cos\phi - mg \\ I_{xx}\ddot{\psi} &= u_\phi \\ I_{yy}\ddot{\theta} &= u_\theta \\ I_{zz}\ddot{\phi} &= u_\psi \end{aligned} \quad (9)$$

Then the corresponding the control input becomes a reduced one such that T the reduced control input matrix is defined as

$$T = \begin{bmatrix} f_{th} \\ T_\phi \\ T_\theta \\ T_\psi \end{bmatrix} = \begin{bmatrix} f_1 + f_2 + f_3 + f_4 \\ C_t(f_1 - f_2 + f_3 - f_4) \\ (f_3 - f_1)l \\ (f_2 - f_4)l \end{bmatrix} \quad (10)$$

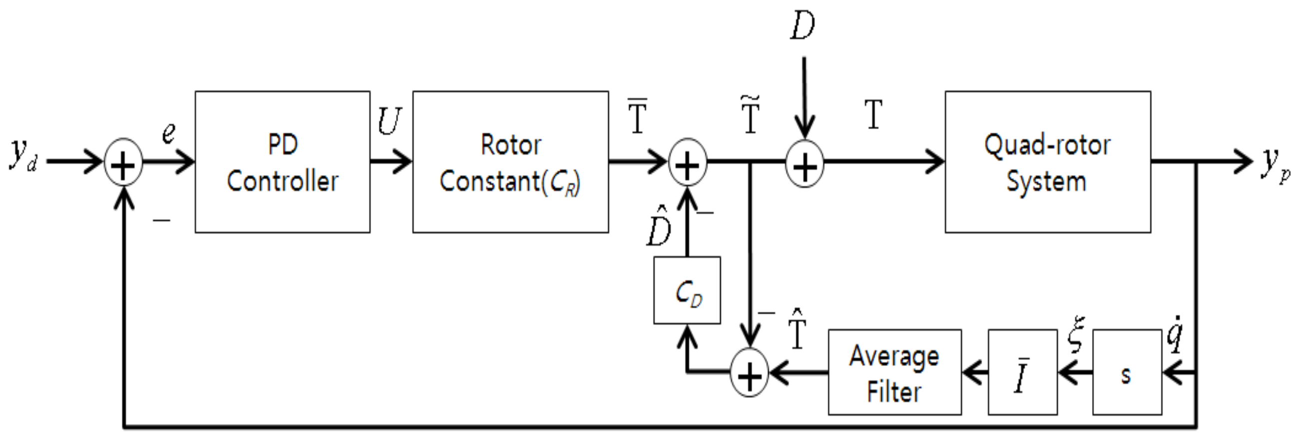

Fig. 3. AbDOB Structure for a Quad-rotor system

B. System Parameters

It is important to know reasonable system parameters to design controllers for an actual system. Especially the moment of inertia values and rotor-force constants are important to estimate disturbance in the AbDOB structure. A system body is described as 5 spheres to obtain values of moment of inertia as shown in Fig. 2.

Fig. 2 shows the real quad-rotor system used for experimental studies of identifying parameters of the system. The corresponding parameter values of the system are listed in Table 1. The moment of inertia about x and y axis are assumed to be same and calculated as I_c. The values between the calculated torque and the measured torque obtained from experiments by gyro differentiation are compared to adjust the value of the moment of inertia. The constant value related between the controller output and the force C_r is obtained by experimental studies of obtaining thrust values using a force sensor [23].

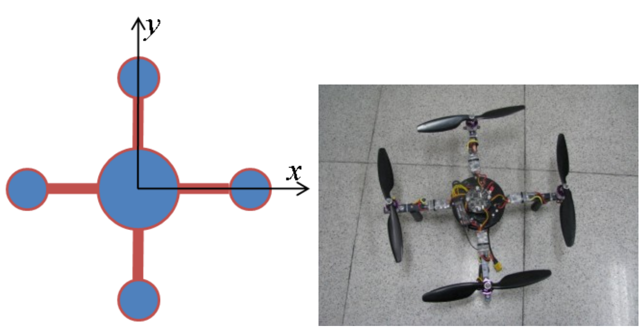

Fig. 2 Quad-rotor system

Table 1 System Parameters

Parameters	Values
Distance between rotor and COG (l)	0.2 m
Moment of Inertia about x, y axis (I_c)	0.004 Kg·m
Rotor-Force Constant (C_r)	0.2

III. Control Schemes

In this paper, a PD control method is used for the altitude control of the quad-rotor system. From equation (9), we can describe the thrust force of each rotor from the control input to the angle control.

$$\begin{aligned} f_1 &= C_r(u_{th} - u_\theta - u_\psi) \\ f_2 &= C_r(u_{th} + u_\phi + u_\psi) \\ f_3 &= C_r(u_{th} + u_\theta - u_\psi) \\ f_4 &= C_r(u_{th} - u_\phi + u_\psi) \end{aligned} \tag{11}$$

where C_r is conversion factor from reaction torque to force.

A. PD Controller

Each control input to the angle control is defined as

$$\begin{aligned} u_\phi &= k_{p\phi}(\phi_d - \phi) + k_{d\phi}(\dot{\phi}_d - \dot{\phi}) \\ u_\theta &= k_{p\theta}(\theta_d - \theta) + k_{d\theta}(\dot{\theta}_d - \dot{\theta}) \\ u_\psi &= k_{p\psi}\psi_{rc} + k_{d\psi}(\dot{\psi}_d - \dot{\psi}) \end{aligned} \tag{12}$$

where $k_{p\phi}, k_{d\phi}$, $k_{p\theta}, k_{d\theta}$ and $k_{p\psi}, k_{d\psi}$ are PD control gains for the roll, pitch and yaw angle control. ψ_{rc} is an input value of the radio frequency remote controller. Since there is no reference sensor to measure the yaw angle, yaw control depends on signal from a remote controller and gyro rate.

The PID control method is used for the thrust control input based on the altitude measurement.

$$u_{th} = m(u_z + g)\frac{1}{\cos\theta\cos\phi} \tag{13}$$

$$u_z = k_{pz}(z_d - z) + k_{dz}(\dot{z}_d - \dot{z}) \tag{14}$$

where k_{pz}, k_{dz}, k_{iz} are PID controller gains and is the altitude.

The nominal PD control input $\bar{T}$ becomes

$$\overline{T} = C_R U \tag{15}$$

where $U = [u_\phi, u_\theta, u_\psi, u_{th}]^T$.

B. Acceleration-based Disturbance Observer

In the framework of PD control, an acceleration-based disturbance observer (AbDOB) is designed as shown in Fig.3. To measure acceleration, angular velocity from the gyro sensor is measured a priori. The angular velocity is differentiated to obtain the angular acceleration.

Now the control input torque values, $\hat{T}$ can be obtained by multiplying the angular acceleration with the moment of inertia such as

$$\hat{T} = \hat{I}\xi \tag{16}$$

where $\hat{I} \in R^{4\times4}$ is the matrix of moment of inertia and $\xi \in R^{4\times1}$ is the angular acceleration vector for roll, pitch and yaw. The difference between $\hat{T}$ and $\tilde{T}$ estimates the disturbance, $\hat{D}$. The estimated disturbance, $\hat{D}$ is subtracted from the nominal control input, $\overline{T}$ to cancel the real disturbance, D.

$$\begin{aligned} T &= D + \tilde{T} \\ &= D + \overline{T} - \hat{D} \end{aligned} \tag{17}$$

If the disturbance estimation is perfect, then disturbance free control input signal can be obtained

$$T = \overline{T} \tag{18}$$

In a practical point of view, estimating the disturbance, $\hat{D}$ is done with an average filter with 4-samples because the estimated torque is too noisy. A constant gain, C_D, is found to adjust the scale of the disturbance estimation, $\hat{D}$ since the value has effect on the stabilization of the system.

IV. Experimental Results

A. Experimental setup

In the experimental studies, AbDOB is applied to roll and pitch axis only. Thus, the estimated control input from the sensor measurement, $\hat{T} = [\hat{T}_\phi, \hat{T}_\theta, 0, 0]^T$.

First, the moment of inertia values are identified by an empirical approach. Fig.4 shows a test bed with a 0.04Kg weight hanging on the right side of the quad-rotor frame, which is considered as a disturbance. The weight is dropped 5 times to give intentional disturbance to the system. The angular motion of the x-axis is tested. Specifications of the test-bed system are listed in Table.2.

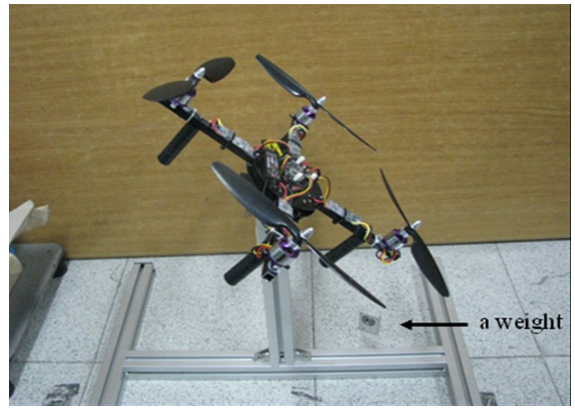

Fig. 4 Test-bed

Table 2 Specifications of System

Variables	Values
Mass	0.95*Kg*
Maximum thrust	2*Kg*
MCU	ATMEGA 644 8 bit processor
Sampling time	4 *msec*

The overall control hardware structure is shown in Fig. 5.

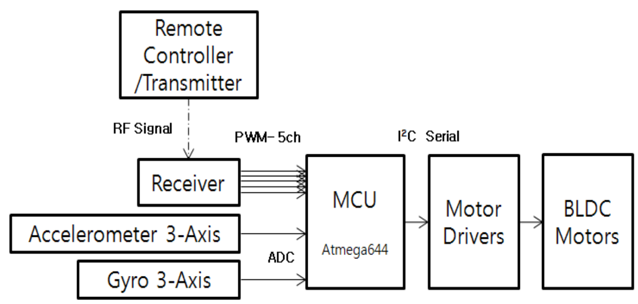

Fig. 5 Block Diagram of Hardware

Table 3 Control Gains

PD Control	K_P	K_D
Roll ϕ	1.8	0.2
Pitch θ	1.8	0.2
Yaw ψ	0.8	0.5
DOB	C_D	
	0.5	

B. PD Control

In order to evaluate the performance of AbDOB, experimental studies are performed without AbDOB first. The PD controller gains for each angle control are listed in Table 3.

The first plot of Fig. 6 shows that the system is rotated about 5° by external force when the disturbance is applied. The measured torque obtained from differentiation of gyro rate is plotted in the second of Fig. 6 and the calculated torque from the control output is plotted last. PD control output torque tries to respond against the disturbances.

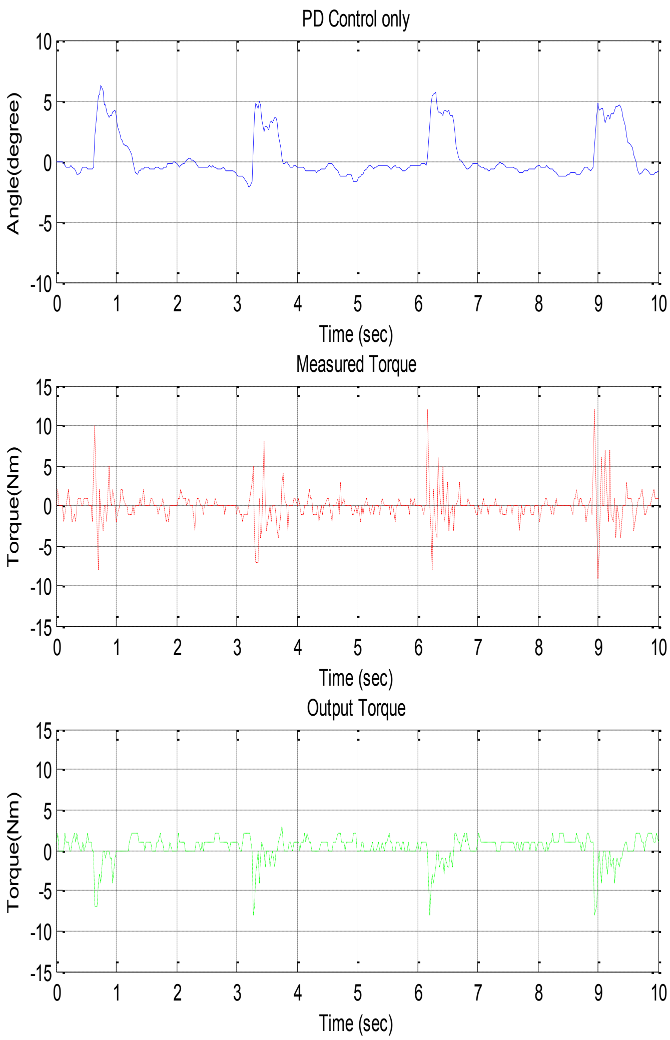

Fig. 6 Attitude control by PD Controller

C. PD Control with AbDOB

The next experiment is carried out with turning AbDOB on. The resultant plot shows that the effect by the applied is reduced by a factor of 1/2 as shown in Fig. 7. The output torque as the third plot of Fig. 7 is bigger than that of the previous case of Fig. 6 because AbDOB tries to compensate for the disturbances.

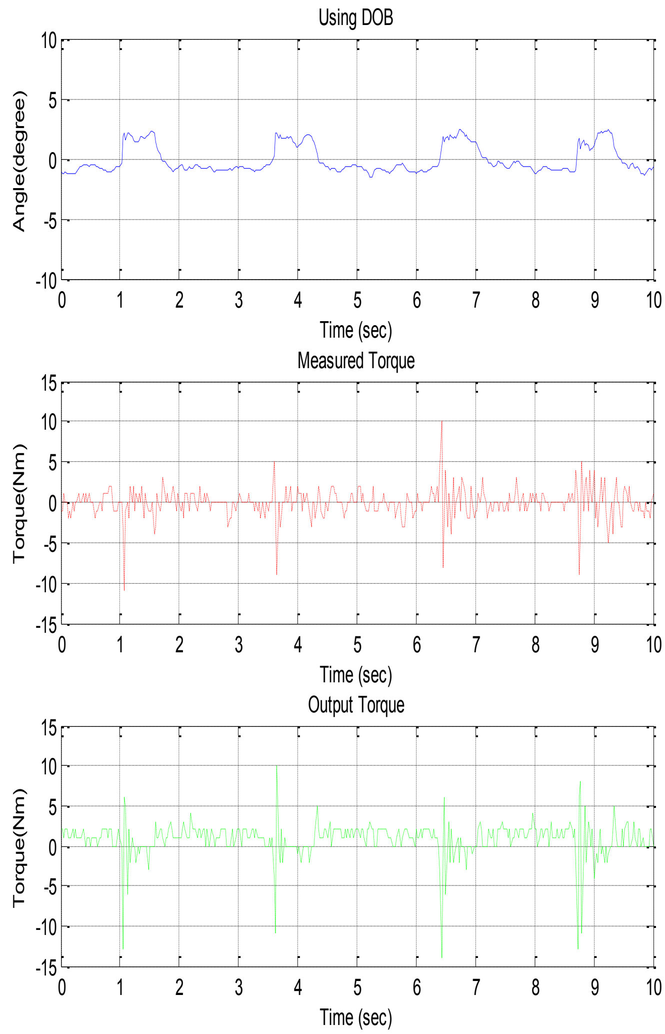

Fig. 7 Attitude control by using AbDOB

D. Flying Test

Lastly, real flying test is performed. Fig.8 shows outdoor flying test using AbDOB. Although it is not clear to demonstrate the better performance by AbDOB, we can prove that stable attitude control has been achieved.

Fig. 8 Flying test result in outdoor

V. Conclusion

To improve the attitude control performance under the presence of disturbance, an acceleration-based disturbance observer is designed and implemented for the practical flying test. The proposed AbDOB structure is simple, but performs better that that of PD controller. There are several issues of improving the attitude control performance using the AbDOB control structure. Firstly, the moment of inertia value should be identified correctly and close to the true value. Here we estimated the value by empirical studies using a test-bed. Secondly, the estimation of acceleration signal using sensors is required to be accurately estimated to estimate the control input torque. Lastly, the disturbance estimation can be improved by selecting appropriate filters.

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